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II. **Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philahelphia, Pa.**

General expressions for the sides of duplicate right-triangles having the same hypotenuse are

$x=(p^2-q^2)(r^2-s^2)+4pqrs,\ y=2rs(p^2-q^2)-2pq(r^2-s^2),$

$z=(r^2-s^2)(p^2+q^2),\ w=2rs(p^2+q^2).$

$\therefore x^2+y^2=z^2+w^2=[(r^2+s^2)(p^2+q^2)]^2$, Let $p=2,\ q=1$.

$\therefore x=3(r^2-s^2)+8rs,\ y=6rs-4(r^2-s^2),\ z=5(r^2-s^2),\ w=10rs.$

$x^2+y^2=z^2+w^2=[5(r^2+s^2)]^2,\ x^2-w^2=z^2-y^2=9r^4+48r^3s-54r^2s^2-48rs^3+9s^4=\square$. This is a square when $r=\frac{17}{12}s$.

$$\therefore x=\frac{689s^2}{64},\ y=\frac{161s^2}{36},\ z=\frac{725s^2}{144},\ w=\frac{85s^2}{6}.$$

$\therefore x=2067s^2,\ y=644s^2,\ z=725s^2,\ w=2040s^2.$

$x^2+y^2=z^2+w^2=(2165s^2)^2=a^2,\ x^2-w^2=z^2-y^2=(333s^2)^2=b^2.$

A solution of this problem is given in J. D. Williams' *Algebra*, page 419. He starts with $a^2=b^2+f^2=c^2+e^2$, and $b^2-c^2=d^2=e^2-f^2$. Then he is to make a^2-b^2 a square, a^2-c^2 a square, and b^2-f^2 a square. He assumes $a^2=(p^2+q^2)(r^2+s^2)$, $b=pr\pm qs$, $c=ps\pm qr$. Then he assumes $r=pm-qn$, $s=pn+qm$. He finally arrives at the conclusion that $a=697$, $b=680$, $f=153$, $c=672$, $e=185$, a set of erroneous values, as Dr. Zerr has pointed out. It is likely that Williams' solution may be carried out so that a set of correct values may be obtained. Williams proposed this problem in 1832 as a challenge problem to the mathematicians of the United States. ED. F.

144. Proposed by JOHN D. WILLIAMS (being the ninth of his 14 challenge problems proposed in 1832).

Make $(m^2+n^2)^2x^2\pm(m^2+n^2)x=\square$, $(m^2-n^2)^2x^2\pm(m^2-n^2)x=\square$, and $4m^2n^2x^2\pm2mnx=\square$.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

Let $m^2+n^2=p,\ m^2-n^2=q,\ 2mn=r.$

$\therefore p^2x^2\pm px=\square,\ q^2x^2\pm qx=\square,\ r^2x^2\pm rx=\square\ldots(1,\ 2,\ 3).$

Let $p^2x^2+px=a^2x^2;\ \therefore x=p/(a^2-p^2).$

This value of x in (2) and (3) gives

$$q^2p^2+qp(a^2-p^2)=\square,\quad r^2p^2+rp(a^2-p^2)=\square\ldots(4,\ 5).$$

Let $q^2p^2+qp(a^2-p^2)=[pq-b(a-p)]^2.$

$\therefore (a-p)=\dfrac{2pq(b+p)}{b^2-pq}$. This value of a in (5) gives

$$r^2b^4+4b^3pqr+2b^2pqr(2p+2q-r)+4bp^2q^2r+p^2q^2r^2=\square$$
$$=(rb+2bpq-pqr)^2,\ \text{suppose.}$$

$$\therefore b=\frac{2pqr}{pr+qr-pq};\ x=\frac{p}{a^2-p^2}=\frac{(b^2-pq)^2}{4bpq\ (b+p)\ (b+q)}.$$

$$\therefore x=\frac{-[(pr+qr-pq)^2-4pqr^2]^2}{8pqr(pr+qr-pq)(pq-pr+qr)(pq+pr-qr)}.$$

$$\therefore x = \frac{-[(4m^3n-m^4+n^4)^2-16m^2n^2(m^4-n^4)]^2}{16mn(m^4-n^4)(4m^3n-m^4+n^4)(m^4-n^4-4mn^3)(m^4-n^4+4mn^3)}$$

$$=\pm\frac{A^2}{16mn(m^4-n^4)B}, \text{ suppose.}$$

x is $\pm$ according as px, qx, rx is $\mp$.

$\therefore (m^2+n^2)x^2 \pm (m^2+n^2)x$

$$=\left[\frac{A}{16mn(m^2-n^2)B}\right]^2[16m^2n^4(n^2-2m^2)+(8mn^3+n^4-m^4)(m^4-n^4)]^2. (6).$$

$(m^2-n^2)x^2 \pm (m^2-n^2)x$

$$=\left[\frac{A}{16mn(m^2+n^2)B}\right]^2[16m^2n^4(n^2+2m^2)-(8mn^3+m^4-n^4)(m^4-n^4)]^2. (7).$$

$4m^2n^2x^2 \pm 2mnx$

$$=\left[\frac{A}{8(m^4-n^4)B}\right][3(m^4-n^4)^2-8m^3n(m^4-n^4)-16m^2n^6]^2. (8).$$

m and n can have any values that make B positive. Let $m=2$, $n=1$; $A=671$, $B=2737$.

(6) gives $(m^2+n^2)^2x^2 \pm (m^2+n^2)x=(290543/262752)^2$.

(7) gives $(m^2-n^2)^2x^2 \pm (m^2-n^2)x=(74481/437920)^2$.

(8) gives $4m^2n^2x^2) \pm 2mnx=(234179/328440)^2$.

AVERAGE AND PROBABILITY.

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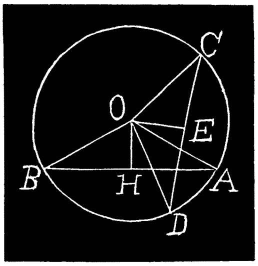
191. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Two random lines cut a given circle. What is the chance that they intersect within the circle?

II. Solulion by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.



Let AB, CD be the random lines, $\angle AOH=\theta$, $\angle COE=\phi$, $\angle EOH=\psi$.

The limits of θ are 0 and $\frac{1}{2}\pi$; of ϕ, 0 and θ; of ψ, $\theta-\phi$ and $\theta+\phi$ for favorable cases, and 0 and π for total cases.

Hence the chance is